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# Mechanical analysis on rock-support interaction in tunnels

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## ABSTRACT

The paper presents an analytical model for the interaction between the tunnel ground and the installed support system. Firstly, an analytical solution for the problem of stress-displacement around a circular tunnel in an elasto-perfectly-plastic medium is introduced. Shotcrete and steel sets are treated as the composite section of a straight beam as a homogenized section of equivalent mechanical properties of the two materials, and the contribution of the supporting elements, including rock bolt, is incorporated into the solution as uniformly distributed radial pressures. Then, the rock-support interaction is expressed with the line branching off from the non-supported relation in the fictitious radial pressure $\sigma_{ra}$ – radial displacement $u_a$ space. Generally, the initial stress in the ground is not hydrostatic, and the support elements are never provided all around the circumference. Secondly, a procedure for calculation of $u_a$ using the displacements of tunnel surface is presented. This can be used for the case in which wall displacement is not evenly distributed. This procedure is examined using FEM simulation of not fully supported simulation.

## 1. INTRODUCTION

The stability of tunnel by the conventional excavation is treated based on a thought whereby the ground surrounding an underground opening becomes a load bearing structural component. The thought considers the activation of the support load of the ground and the interaction with the installed support system. The activation will result in a minimization of load supported by the artificial support elements, such as shotcrete (SC), steel sets (SS) and rockbolt (RB). The mechanical concept of this method is characterized by Ground Reaction Curve (GRC, or Ground Characteristic Curve, GCC) and the contribution of the support system. The tunnelling method has been used in the wide range of ground conditions, soft or hard, and shallow or deep. The problem of assessing the mechanical stability of tunnel ground itself and the interaction with the

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support elements is an important geotechnical problem with

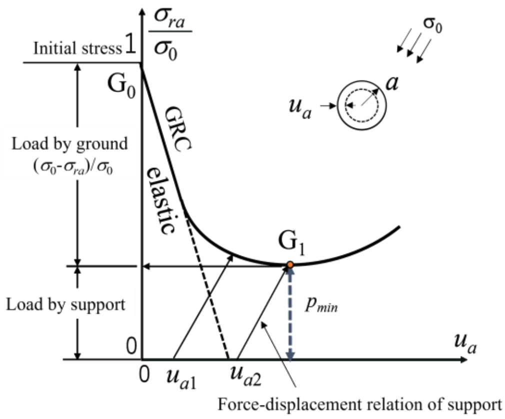



Fig. 1 Schematic representation of ground reaction curve turning upward.

practical applications in the field of civil engineering. If the state of stress exceeds the strength of the ground due to the stress redistribution, yielding may occur in the vicinity of the tunnel opening. To prevent large deformation and the collapse in the ground during construction, support pressure must be applied to establish the load bearing zone within the ground in the vicinity of the opening.

Fig. 1 shows a schematic representation of ground reaction curve, and it is often said that the optimization postulates the curve turning upward (Rabcewicz, 1969, JSCE, 2016). The abscissa is the relative radial displacement and the ordinate the lining pressure. This figure also shows that the lining must be installed at the timing to meet at the minimum point of the GRC, not too early and not too late, to avoid the increase in the load to the support. The stress state around tunnel face can be expressed in the three-dimensions, representing major, intermediate, and minor principal stresses. Excavation-induced stresses shown in Fig. 1 has been primarily restricted to two-dimensions and the axial symmetry of both the geometry and the stresses is assumed. Numerical studies on GRC have been reported, e.g. Sezaki et. al. (1994), Nishimura (1991), and such solutions incorporate the effect of gravity and the strain softening behaviour of rocks, nevertheless, the solutions to explain the interaction between the ground and the supporting elements is never developed (Kovari, 1993).

This paper presents an analytical model for the interaction between the tunnel ground and the installed support system under hydrostatic condition. This model is based on the solution of stress-displacement around a circular tunnel in an elasto-perfectly-plastic medium and incorporates the contribution of the supporting elements as uniformly distributed radial pressures. This solution expresses the effect of the supporting elements with the line branching off from the non-supported relation in the fictitious radial pressure $\sigma_{ra}$ – radial displacement $u_a$ space. Generally, the initial stress in the ground is not hydrostatic, and the support elements are never provided all around the circumference. Therefore, for practical applications of this model, including partially supported case, we need a procedure to calculate the outputs, radial displacement and pressures using measured displacements and stresses. This paper proposes a numerical technique to

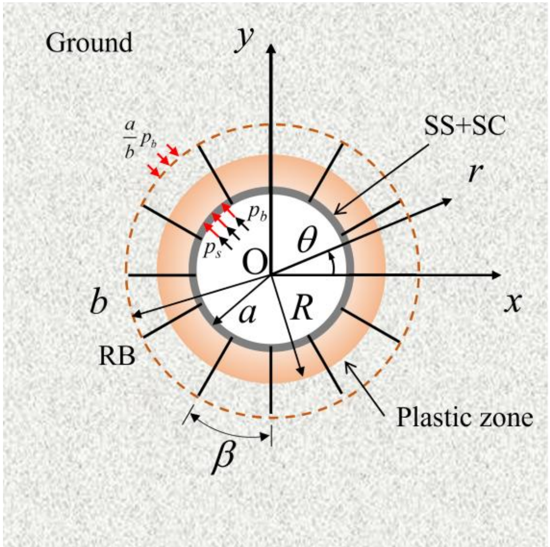



Fig. 2 Analytical model of circular tunnel with rock bolt, shotcrete and steel sets.

obtain a representative value of $u_a$ as the change in the area of tunnel cross section. A series of FEM simulations is conducted for circular tunnel opening with the consideration for the extent of the supports in circumferential direction, fully and partially covered cases. The result of this numerical trial will be summarized in the relation of the incremental displacement after the support elements installed and the radial pressure acting on the circular surface.

## 2. ANALYTICAL SOLUTION FOR 'FULL-CLOSED' SUPPORTED CASE

### *2.1 Summary of the modeling for tunnel excavation*

GRC is a characteristic line that records the decrease of an apparent (fictitious) internal (radial) support pressure, from the *in-situ* pressure to zero. This pressure reflects the tunnel excavation process as the tunnel is being excavated past the section of interest and continues to be excavated beyond the reference position (usually the location of the tunnel face). The internal pressure ($\sigma_{ra}$) acts radially on the tunnel profile (from the inside) and represents the support resistance needed to hinder any further displacement at that specific location. In reality, this pressure represents an idealized sum of the contribution of the nearby unexcavated tunnel core (surrounding rock mass) and the support installed, and $\sigma_{ra}$=0 means a fully excavated non-supported tunnel. GRC depends on the rock mass behavior, and it is assumed to be linear for an elastic material, but it varies if the material is elasto-plastic or visco-elastic etc.. In this paper, GRC is presented as an analytical solution for the problem of stress-displacement around the tunnel in an elasto-perfectly-plastic medium. Displacements are computed from the integration of strains, which are decomposed into elastic and plastic components. A plastic potential will give the plastic component.

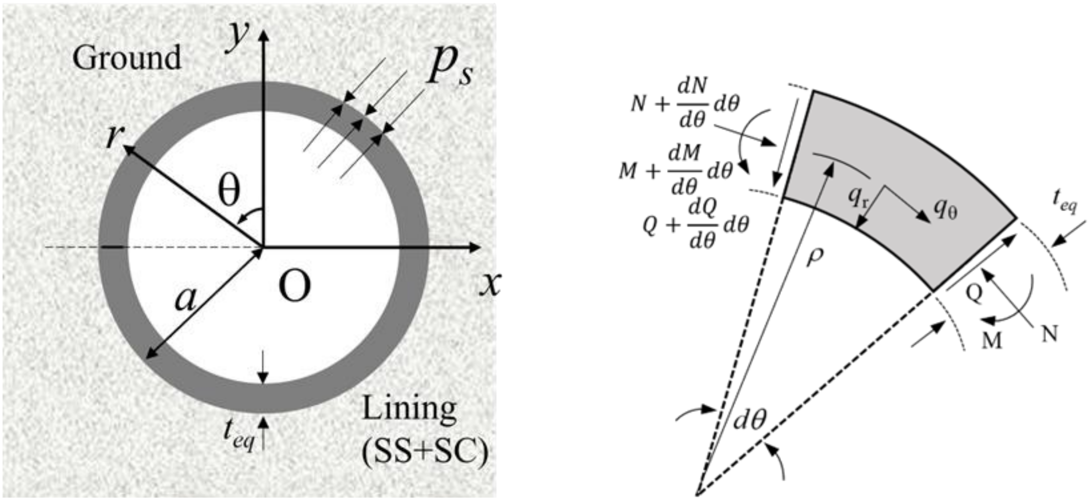
In this paper, a mechanical modelling for shotcrete (SC) and steel sets (SS) is introduced and the model consists in treating the composite section of a straight beam as a homogenized section of equivalent mechanical properties of the two different materials. The contribution of bolt (RB) is modelled as uniformly distributed pressure, then the effect of the supporting elements can be expressed with the line branching off from the relation of non-supported ground reaction curve. The line will give a preliminary estimation for the load shared by the ground and the supports. Numerical techniques such as FEM simulation will represent the convergence confinement for the case of the extent of the support in circumferential direction and for non-circular tunnel shapes. Significantly less computational resources are required compared to the numerical results; hence, this analytical method is well suited at the preliminary stage of a project to conduct extensive sensitivity analyses and identify critical support scenarios.

*2.2 Model setup for analytical solution*

The analytical model described here specifically allows an improved understanding of the role of variables involved in the problem. Each variable acts as a part on the resulting overall stability of the opening. A cylindrical cavity of radius, *a*, is excavated in ground. A Cartesian coordinate system, (*x*, *y*), is considered to have the origin at the center of the cavity (point O in Fig. 2). Fig. 2 shows a cylindrical cavity of radius, *a*, in a media subjected to the uniform compression $\sigma_{x0}=\sigma_{y0}=\sigma_0$. This is the model to have an analytical solution for assessing the stability of ground due to tunnel excavation. Several methods have been developed to address the problem of stability of underground excavation. The initial (or in-situ) vertical stress, $\sigma_{y0}$, is considered to be lithostatic, i.e., $\sigma_{y0}=\gamma(h\text{-}y)$ and the initial (or in-situ) horizontal stress, $\sigma_{x0}$, is obtained by multiplying the coefficient of earth pressure at rest, $K_0$, and the initial vertical stress, i.e., $\sigma_{x0}= K_0\sigma_{y0}$. Under this initial stress condition, it is obvious that the plastic area is never developed with circular shape and the equilibrium in force can be equated with both in radial direction and tangential directions. To avoid the complex formulation, the simple modeling for the initial stress state, i.e., the isotropic condition in infinite media, is assumed in this study as seen in Fig. 2. This media is assumed to be elastic-perfectly-plastic material expressed with the Mohr-Coulomb failure criterion and the state of stress in the circle of radius *R* is expressed by the failure criterion. The stress field inside the circle of radius *R* represented in Fig. 2 is compatible with a plastic state which is expressed with the Mohr-Coulomb failure criterion while the stress field outside the circle is compatible with an elastic state which is defined by the isotropic stress field. Inside the plastic circle, the stresses are assumed to be redistributed due to stress release to explain the progress of excavation in the cavity of radius *a*. Note that in the plastic circle, radial stress, $\sigma_r$, and tangential (or hoop) stress, $\sigma_\theta$, are assumed to be the principal stresses, nevertheless accounting the self-weight under gravity field. The contribution of bolt (RB) is modeled as uniformly distributed pressure on the tunnel wall and the end of bolt.

*2.3 Analytical solution for non-supported circular tunnel*

Considering a polar coordinate system, $(r,\theta)$, the force equilibrium equations inside the plastic circle, neglecting the self-weight of the ground material, can be derived as a differential equation. For the radial direction, the equation is:




(a) (b)

Fig. 3 (a) Schematic representation of infinite cylinder-shaped beam subjected to a uniform radial pressure $p_s$, (b) the balance of bending moment, thrust and shear forces along the stretch of arch.

$$\frac{d\sigma_r}{dr} + \frac{(1-\zeta)\sigma_r - S_c}{r} = 0 \tag{1}$$

where $\zeta=(1+\sin\phi)/(1-\sin\phi)$, $S_c=2c\cos\phi/(1-\sin\phi)$, $c$ is the cohesion and $\phi$ is the friction angle. The Mohr-Coulomb failure criterion is expressed in terms of principal stresses, assuming that radial stress, $\sigma_r$, and tangential (or hoop) stress, $\sigma_\theta$, are the principal stresses. Solving Eq. (1), the solution of $\sigma_r$ only is obtained for $\zeta>1$ ($\phi>0$):

$$\sigma_r = \frac{S_c}{\zeta - 1}\left\{\left(\frac{r}{a}\right)^{\zeta-1} - 1\right\} + \sigma_{ra}\left(\frac{r}{a}\right)^{\zeta-1} \tag{2}$$

Considering the following boundary condition: $\sigma_r=\sigma_{ra}$ at $r=a$, this equation can be rewritten:

$$\sigma_{ra} = \left(\sigma_{rR} + \frac{S_c}{\zeta-1}\right)\left(\frac{R}{a}\right)^{1-\zeta} - \frac{S_c}{\zeta-1} \tag{3}$$

where the stress in radial direction on $r=R$ (plastic radius), $\sigma_\theta + \sigma_{rR} = 2\sigma_0$ and $\sigma_\theta = \zeta\sigma_{rR} + S_c$. The radius of plastic circle is given as follows:

$$\frac{R}{a} = \left\{\frac{(\zeta-1)\sigma_{rR} + S_c}{(\zeta-1)\sigma_{ra} + S_c}\right\}^{\frac{1}{\zeta-1}} \tag{4}$$

Displacements are computed from the integration of strain. Note that due to the symmetry of the problem, the tangential displacements are always zero. Assuming small deformation analysis, the increments of strain are decomposed into elastic and plastic. In the plastic area, the material must obey the yield condition. The Mohr-coulomb criterion in two-dimension is introduced to evaluate the stress state in the plastic area and a plastic potential can be given the following form:

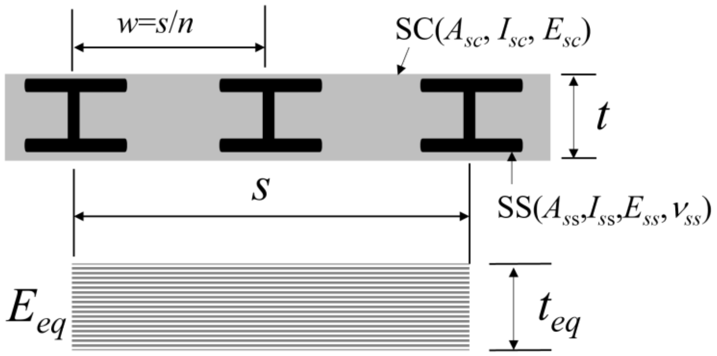



Fig. 4 Schematic representation of a section of liner consisting of different materials SC and SS and an equivalent section for the composite elastic liner.

$$G(\sigma_r, \sigma_\theta) = \sigma_\theta - \frac{1+\sin\psi}{1-\sin\psi}\sigma_r \tag{5}$$

where $\psi$ is the dilation angle and $\psi=\phi$ means the associated flow rule. A flow rule gives the plastic strains, then, one gets the following expression:

$$\frac{1+\sin\psi}{1-\sin\psi}\varepsilon_\theta{}^p + \varepsilon_r{}^p = 0 \tag{6}$$

Rewriting Eq. (6) as a function of displacements, the following is obtained.

$$\frac{\partial u_r}{\partial r} + N_\psi \frac{u_r}{r} = 0 \tag{7}$$

where $N_\psi$=(1+sin$\psi$)/(1-sin$\psi$). With the condition of displacement on $r=R$, the radial displacement in the plastic area is given:

$$u_r = -\frac{1+\nu}{E}(\sigma_0 - \sigma_{rR})\left\{\frac{a^2}{r} + \frac{R^{N_\psi - 1}}{r^{N_\psi}}(R^2 - a^2)\right\} \tag{8}$$

where $E$: Young's Modulus, $\nu$: Poisson's ratio of the ground material. Excavation of tunnel is modeled with a stepwise release of stress $\sigma_{ra}$ on the tunnel circular envelop from its initial value $\sigma_{ra}{}^0=\sigma_0$ and $K_0$=1. The stress release can be defined as $\lambda=(\sigma_{ra}{}^0-\sigma_{ra})/\sigma_{ra}{}^0$, where the initial state $\lambda$=0 and the fully excavated $\lambda$=1.

*2.4 Analytical solution for support-tunnel interaction*

This section describes a methodology for the mechanical model of the composite liner consisting of shotcrete and steel sets as illustrated Fig. 3(a). The modeling consists in treating the composite section of a straight beam as a homogenized section of equivalent mechanical properties and subjected to pressure acting on the outer surface as seen in Fig. 3(a). Fig. 3(b) shows a circular arch of thickness $t_{eq}$. This figure represents the balance of bending moment, thrust and shear forces along the stretch of arch of differential length $rd\theta$. The equations of the equilibrium are (Flügge, 1967, Carranza-Torres and Diederichs, 2009):

$$\frac{dQ}{d\theta} - N + p_r R = 0, \frac{dN}{d\theta} + Q + p_\theta R = 0, \frac{dM}{d\theta} + QR = 0 \tag{9}$$

where $N$, $Q$ and $M$ represent thrust, shear force and bending moment, and expressed as

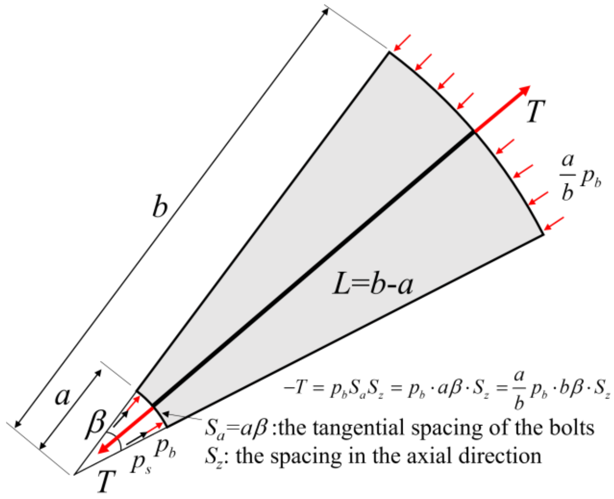



Fig. 5 A uniformly distributed pressure at the tunnel opening and at the end of the bolt, equivalent to the tension *T* of the bolt.

follows:

$$N = -\frac{D}{R}\cdot\left(u_r + \frac{du_\theta}{d\theta}\right) - \frac{K}{R^3}\cdot\left(u_r + \frac{d^2u_r}{d\theta^2}\right), \quad M = -\frac{K}{R^2}\cdot\left(u_r + \frac{d^2u_r}{d\theta^2}\right)$$

In this expression, $K=EI$, $D=EA$ for plane stress condition and $K=EI/(1-\nu^2)$, $D=EA/(1-\nu^2)$ for plane strain condition, $I$: moment of inertia of area, $A$: area of cross section. The characteristic of the arch can be expressed in terms of non-dimensional parameter $\xi = D\rho^2/K$. For a rectangular section of arch of radius $\rho$, thickness $t$ and width $w$, the area is $wt$ and the moment of inertia is $I=wt^3/12$, then $\xi=12/(t/\rho)^2$. For the smaller value of $t/\rho$, this may represent the case of thin liner and Eq. (9) is

$$N = -\frac{D}{R}\cdot\left(u_r + \frac{du_\theta}{d\theta}\right), \quad M = -\frac{K}{R^2}\cdot\left(\frac{d^2u_r}{d\theta^2}\right) \tag{10}$$

Fig. 4 shows a schematic representation of a section of liner consisting of different two materials SC and SS, and the equivalent section for the composite liner. The thickness $t_{eq}$ and the equivalent Young's modulus $E_{eq}$ are:

$$t_{eq} = \sqrt{\frac{12(K_1+K_2)}{D_1+D_2}} \qquad E_{eq} = \frac{n(D_1+D_2)}{t_{eq}\cdot s} \tag{11}$$

where $D_1$, $K_1$ are for shotcrete and $D_2$, $K_2$ are for steel sets. For an infinite cylinder-shaped beam, a uniform radial pressure $p_s$ can be computed as follows:

$$p_s = E_{eq}t_{eq}\frac{u_r\big|_{r=a}^{\lambda_f} - u_r\big|_{r=a}^{\lambda_{in}}}{a^2} \tag{12}$$

where $\lambda_{in}$ is the value of the stress release rate at the support elements installed. Note that Eq. (12) considers that the pressure acting on the composite liner is due to the deformations of the ground after the liner is installed.

The solution of the tunnel ground reinforced by untensioned bolt is obtained with the assumption that the contribution of the bolt can be approximately as uniformly distributed pressure. The tension *T* in the rockbolt is evaluated as follows.

$$T=\left\{\left(u_r\big|_{r=b}^{\lambda_f}-u_r\big|_{r=b}^{\lambda_{in}}\right)-\left(u_r\big|_{r=a}^{\lambda_f}-u_r\big|_{r=a}^{\lambda_{in}}\right)\right\}\frac{E_bA_b}{b-a} \tag{13}$$

where $E_b$ is the Young's modulus and $A_b$ is the area of the cross section, *b* is the radial distance to the tip of the bolt, i.e. the bolt length *L*=*b*-*a* as shown in Fig. 5. At the tunnel opening the support pressure is $p_b=-T/S_aS_z$, where $S_a$ is the tangential spacing of the bolts at the tunnel perimeter, and $S_z$ is the spacing in the axial direction. At the end of the bolt, the pressure is $ap_b/b$, directed inwards. This figure also shows the contribution of the supporting elements as uniformly distributed radial pressures $p_0$ at *r*=*a*, where $p_0$ is the sum of $p_s$ and $p_b$. Then the boundary conditions to solve Eq. (1) are:

1) $\sigma_r\big|_{r=a}=\sigma_{ra}+p_0,\quad p_0=p_b+p_s=\dfrac{-T}{S_aS_z}+p_s,\quad S_a=a\beta$

2) $\sigma_\theta\big|_{r=R}=\zeta\sigma_r+S_c$

3) $\sigma_r\big|_{r=R-dr}=\sigma_r\big|_{r=R+dr}$

4) $u_r\big|_{r=R-dr}=u_r\big|_{r=R+dr}$

5) $\sigma_r\big|_{r=b-dr}-\sigma_r\big|_{r=b+dr}=\dfrac{a}{b}p_b$

6) $\sigma_r\big|_{r\to\infty}=\sigma_0$

The solution of a deep circular tunnel in homogeneous and isotropic medium with Coulomb brittle failure and supported with uniformly distributed pressure at the tunnel opening and at the end of the bolt is given as follows.
The radius of plastic circle:

$$\left(\frac{R}{a}\right)^{\zeta-1}=\frac{(\zeta-1)\left\{2\sigma_0+\dfrac{ap_b}{(1-\nu)b}+\dfrac{2S_c}{\zeta-1}\right\}}{(\zeta+1)\left\{(\zeta-1)(\sigma_{ra}+p_0)+S_c\right\}} \tag{14}$$

The solutions for the stresses and the displacement:
*a*<*r*<*R*

$$\sigma_r=\left(\sigma_{ra}+p_0+\frac{S_c}{\zeta-1}\right)\left(\frac{r}{a}\right)^{\zeta-1}-\frac{S_c}{\zeta-1} \tag{15}$$

$$\sigma_\theta=\zeta\left(\sigma_{ra}+p_0+\frac{S_c}{\zeta-1}\right)\left(\frac{r}{a}\right)^{\zeta-1}-\frac{S_c}{\zeta-1} \tag{16}$$

$$u_r=-\frac{1+\nu}{E}\left\{(1-2\nu)(c_1-\sigma_0)R-\frac{c_2}{R}-(\sigma_0-\sigma_{rR})\left[\frac{a^2}{R}-\frac{a^2}{r}\cdot\left(\frac{r}{R}\right)^{N_\psi}\right]\right\}\left(\frac{R}{r}\right)^{N_\psi} \tag{17}$$

where the constants $c_1$ and $c_2$ are calculated as follows:

$$c_1=\sigma_0+\frac{ap_b}{2(1-\nu)b},\quad c_2=\frac{-1}{\zeta+1}\left[(\zeta-1)\left(\sigma_0+\frac{ap_b}{2(1-\nu)b}\right)+S_c\right]R^2$$

*R*<*r*<*b*

$$\sigma_r=c_1+\frac{c_2}{r^2} \tag{18}$$

$$\sigma_\theta=c_1-\frac{c_2}{r^2} \tag{19}$$

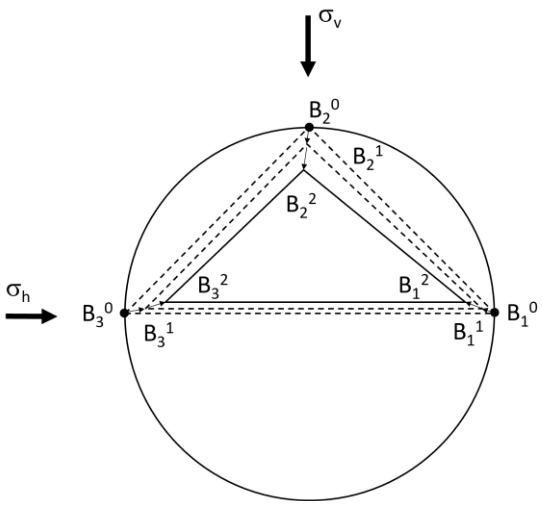



Fig. 6 Three measuring points on tunnel surface and their displacements.

$$u_r = -\frac{1+\nu}{E}\left[c_1(1-2\nu)r - \frac{c_2}{r} - (1-2\nu)\sigma_0 r\right] \tag{20}$$

*b<r*

$$\sigma_r = \sigma_0 - \left(\sigma_0 + \frac{a}{b}p_b - c_1 - \frac{c_2}{b^2}\right)\frac{b^2}{r^2} \tag{21}$$

$$\sigma_\theta = \sigma_0 + \left(\sigma_0 + \frac{a}{b}p_b - c_1 - \frac{c_2}{b^2}\right)\frac{b^2}{r^2} \tag{22}$$

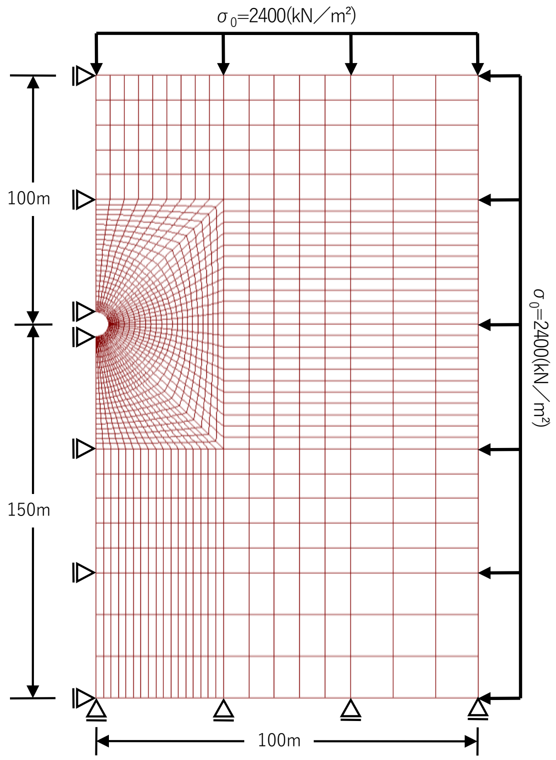
$$u_r = -\frac{1+\nu}{E}\left(\sigma_0 + \frac{a}{b}p_b - c_1 - \frac{c_2}{b^2}\right)\frac{b^2}{r} \tag{23}$$

## 3. REPRESENTATIVE VALUE OF DISPLACEMENT AND NUMERICAL EXAMPLE

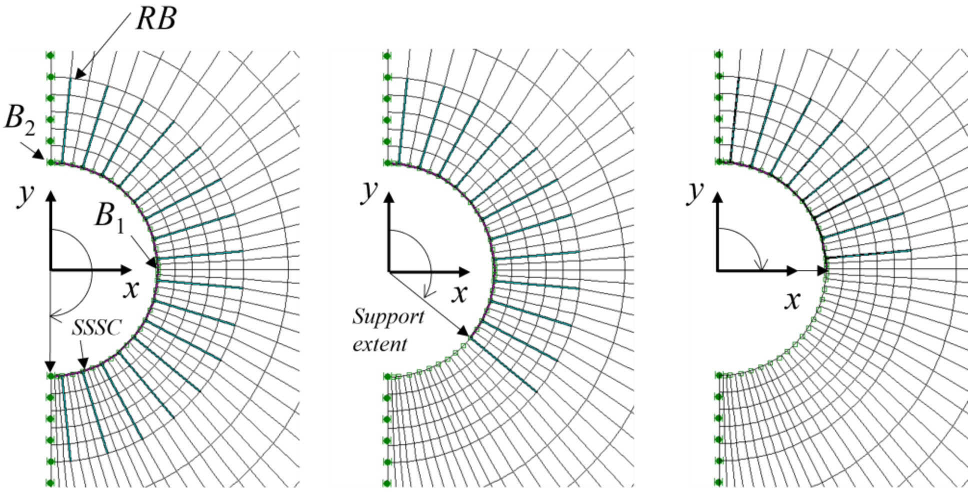
### *3.1 Calculation of representative displacement $u_a$*

As mentioned in chapter 1, the initial stress in ground is generally not hydrostatic and the support elements are never provided all around the tunnel excavation surface. In such cases, to express the effect of the supporting elements with the relation in $\sigma_{ra}$ – $u_a$ space, we need an average value of the radial displacement. However, it can be obvious that not only will a limited amount of data for displacement be given, but data for the pressures are not measured in practical cases. Therefore, for practical applications of this model, procedures are required to evaluate the radial displacement and the pressures as representative values using measured data. This section presents a method to get a representative value of $u_a$ using data obtained at tunnel excavation surface.

Suppose that a uniform radial displacement $u_a$ occurs on the surface of a circular hole of radius *a*. If $u_a$ is very small with respect to *a* and second-order or higher terms are negligible, then the following equation is derived.




(a) Numerical model of a circular tunnel




(b)100 Supported (c)075 Supported (d)050 Supported

Fig. 7 Numerical model of a circular tunnel in an elastic-perfectly plastic ground and supporting element.

$$\frac{\Delta S_{xy}}{S_{xy}} = \frac{2u_a}{a} \tag{24}$$

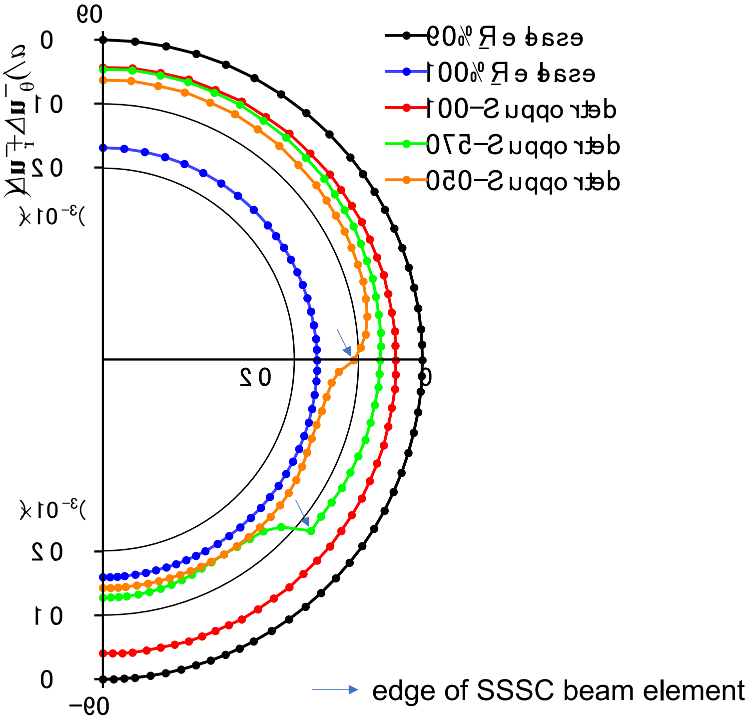



Fig. 8 Incremental displacement of points on the tunnel surface and deformation of tunnel.

where $S_{xy}$ is the area of the tunnel cross section, $\Delta S_{xy}$ is the change in $S_{xy}$ due to tunnel wall displacement. In numerical simulation, the coordinates of the points on the tunnel surface are easily obtained. Then, the cross-section can be treated as a polygon and the area will be easily calculated. At real tunnel excavation site, the number of the measuring point is very limited as seen in Fig. 6. However, if the lengths of the three sides of the triangle are obtained, it will be possible to use and get the value of $u_a$ by Eq. (24) approximately. A series of FEM simulations is conducted for circular tunnel opening, including consideration for the extent of the supports in circumferential direction, fully and partially covered cases. The result of this numerical trial will be summarized in the next section and the possibility of the calculation of $u_a/a$ by Eq. (24) is described.

*3.2 Analytical model of circular tunnel.*

Fig. 7 shows the element division diagram used in the finite element analysis. The circular hole has a radius of $a$=5m, and this is a half-section analysis that utilizes symmetry. In this analysis, RB is modeled as an axial element, and SS and SC are modeled as beam element characterized with $E_{eq}$ and $t_{eq}$. It is assumed that no relative displacement occurs in the circumferential direction at the contact surface between the beam elements and the elements representing the ground. This paper presents three cases for the extent of the support elements as shown in Fig. 7(b), (c) and (d). Fig. 7(b)

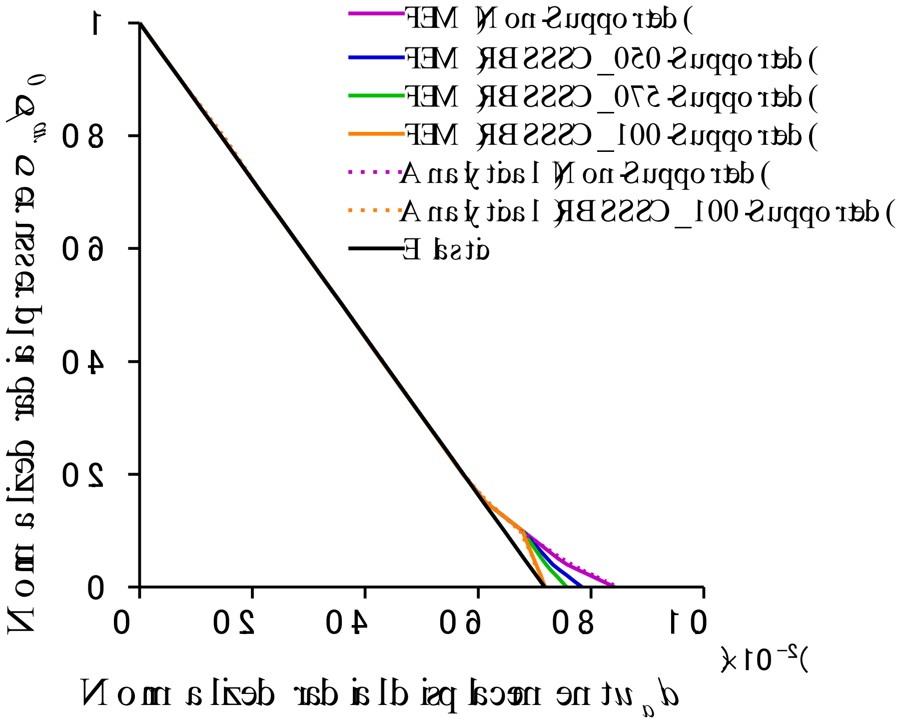



Fig. 9 An example of the numerical simulation for supporting extent: fully and partially installed cases.

displays the model in which beam elements are placed around the entire inner circumference of the circular hole and 16 axial elements are placed at regular interval $\beta$ in the circumferential direction. The number of axial elements is changed to 12 (Fig. 7(c)) and 8 (Fig. 7(d), keeping $\beta$ constant, and the installation range of the beam element is also changed accordingly. In addition to these changes in the installation range, examples of only axial elements and only beam elements are also simulated, making a total of nine installation patterns (excluding the analysis without support). The display format of the analysis cases is as follows. RBSSSC indicates that RB, SS, and SC elements are installed together, RB and SSSC indicate that either RB or a combination of SS and SC are installed, and 100, 075 and 050 indicate the installation range of the support elements. These three numbers do not numerically indicate the exact installation length in the circumferential direction, but these are intended to identify the installation range of the support as approximate values. For example, 'RBSSSC_075' indicates the installation of 12 RB elements and SSSC element up to the range as shown in Fig. 7(c). The properties of the ground are $E_g$=500MPa, $\nu_g$=0.495, $\sigma_0$=2.4MPa, $c$=800MPa, $\phi$=30º and $\psi$=30º. The support elements are assumed to be elastic and characterized as follows. [Rock bolt] Elastic modulus $E_b$=206GPa, diameter $d_b$=24mm, cross-sectional area

Table 1. Values of $u_a/a$ at $\lambda$=1 (fully excavated) obtained by Eq. (24) for the three cases of supporting extent.

| Analytical Case | $u_a/a$ | | |
|---|---|---|---|
| | Eqs. (8), (17) | Calculation 1 | Calculation 2 |
| **Non-Supported** | | | |
| Elastic | 0.00717 | 0.00716 | 0.00733 |
| $c$=800MPa, $\phi$=30º | 0.00848 | 0.00841 | 0.00857 |
| **Supported** (c=800MPa, $\phi$=30º) | | | |
| RBSSSC_100 | 0.00720 | 0.00719 | 0.00734 |
| SSSC_100 | 0.00724 | 0.00725 | 0.00740 |
| RB_100 | 0.00798 | 0.00787 | 0.00814 |
| RBSSSC_075 | - | 0.00757 | 0.00747 |
| SSSC_075 | - | 0.00765 | 0.00755 |
| RB_075 | - | 0.00801 | 0.00818 |
| RBSSSC_050 | - | 0.00783 | 0.00788 |
| SSSC_050 | - | 0.00790 | 0.00796 |
| RB_050 | - | 0.00814 | 0.00829 |

$A_b$=4.52×10$^{-4}$m$^2$, circumferential installation interval $\beta$=11.25° (number of RB installed $m$=32 ($0 \le \theta \le 2\pi$)), installation length $L$=4m.
[Steel support] $E_{ss}$=206GPa, $A_{ss}$=3.97×10$^{-3}$m$^2$, Moment of inertia $I_{ss}$=1.62×10$^{-5}$m$^4$, installation interval $\alpha$=1m.
[Concrete] $E_{sc}$=4000MPa, $A_{sc}$=0.196m$^2$, $I_{sc}$=6.67×10$^{-4}$m$^4$.
Obtaining $K_1=E_{ss}I_{ss}$, $D_1=E_{ss}A_{ss}$, $K_2=E_{sc}I_{sc}$, and $D_2=E_{sc}A_{sc}$ and then substituting them into equation (15), we obtained $E_{eq}$=7552MPa and $t_{eq}$ = 0.21m.

It is obvious that not only the physical properties of the ground but also the physical properties of the support members affect the rock-support mechanical interaction. In this paper, we used the above values, placing emphasis on comparing the Finite Element simulations and the theoretical analysis with the same physical properties. Note that both the Finite Element simulations and the theoretical analysis use an initial stress of $\sigma_0$ = 2400kPa to determine the behavior of the ground and the support elements in response to stress release in the circular hole.

### *3.3 Analytical results and deformation of circular tunnel*

Fig. 8 shows the incremental displacement at tunnel surface in the finite element analysis, starting from $\lambda_{in}$=0.9 and finishing at $\lambda$=1 (fully excavated). These results display the effect of the support installation extent on the reduction of deformation. In the results of '075-supported' and '050-supported' cases, the displacement becomes large beyond the edge of the supports and the amount of displacement around the invert approaches

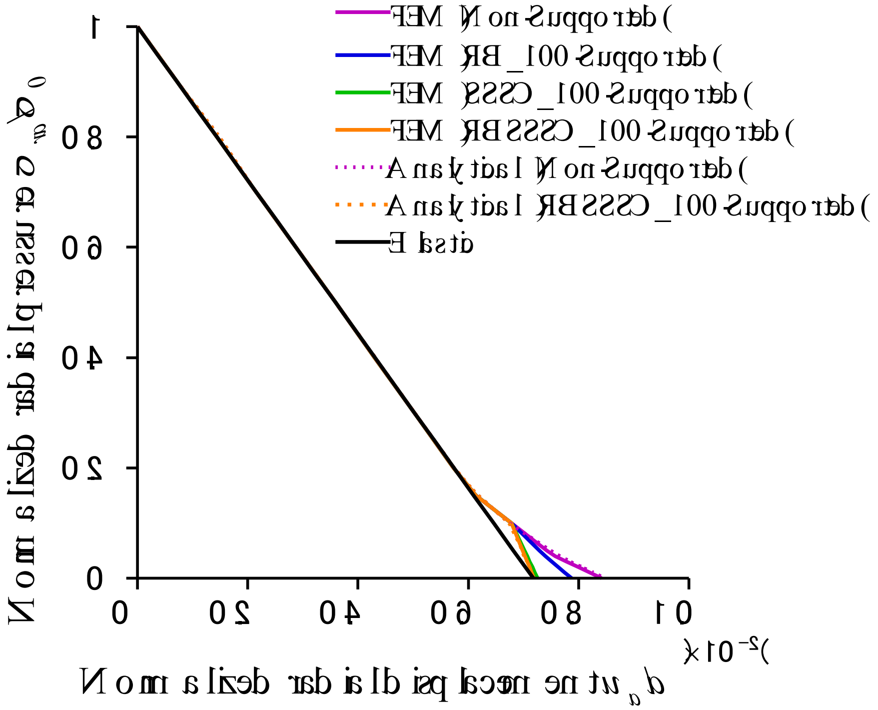


Fig. 10 An example of the numerical simulation for the support element.

the amount of the non-supported case. The value of $u_a$ is calculated in two types: one is the calculation using the coordinates of all nodes as seen in Fig.8 (Calculation 1), and the other is the calculation using the coordinates of $B_0$ and $B_1$ in Fig.7(b) (Calculation 2). Their initial positions are $B_1$ ($x$, $y$) = (5, 0) and $B_2$ ($x$, $y$) = (0, 5) at $\sigma_{ra}=\sigma_0$.

Fig. 9 shows the $\sigma_{ra}$-$u_a/a$ relationship ($\lambda_{in}$=0.9) for the support element installation extent as shown in Fig. 7, including unsupported simulation. The solid line shows the results of the finite element analysis, and the dashed line shows the results of the theoretical analysis described in chapter 2. As explained in chapter 1, the $\sigma_{ra}$-$u_a/a$ relationship after support installation is drawn as the relationship that branches off from that of ground characteristic curve. The theoretical analysis shown by the dashed line expresses well the finite element analysis result shown by solid line. This figure shows the $\sigma_{ra}$-$u_a/a$ relationship focusing on the support installation range for simultaneous installation of RB and SSSC. If the installation range is not the entire circumference, it is impossible to treat it as an axisymmetric problem as seen in Fig. 8. The comparison of the installation extent is made using the expression (24) for a representative value of $u_a/a$. Values of $u_a/a$ at $\sigma_{ra}$=0 ($\lambda$=1.0) are listed in Table 1. It can be concluded that the result by Calculation 1 is nearly the same as the theoretical result, and the result by Calculation 2 is an approximation of the theoretical result.

Fig. 10 shows the $\sigma_{ra}$-$u_a/a$ relationship ($\lambda_{in}$=0.9) for the support element installation range of $0\leq\theta\leq2\pi$, focusing on the type of support element. The $u_a/a$ reduction effect of installing only SSSC is greater than that of installing only RB. Furthermore, the comparison of installing only SSSC with simultaneously installing RB and SSSC (RBSSSC) shows that the reduction in $u_a/a$ in RBSSSC case is smaller than the sum of the individual effect by RB and SSSC simulations. The theoretical analysis in chapter 2 gave the values of $p_s$ and $p_b$ as follows: $p_s/(1-\lambda_{in})\sigma_0$=0.590 and $p_b/(1-\lambda_{in})\sigma_0$=0.124 in RBSSSC_100, $p_s/(1-\lambda_{in})\sigma_0$=0.656 in SSSC_100, $p_b/(1-\lambda_{in})\sigma_0$=0.336 in RB_100. These values indicate the load bearing ratio of the support elements to the load to be supported at the support instillation ($\lambda_{in}$=0.9). $p_b$ is lower in RBSSSC_100 than in RB_100 and this is an example of the mechanical interaction between the support elements and ground.

## 4.CONCLUSIONS

There have been reports related to the ground reaction curve, particularly with the use of improved analytical and numerical models for the prediction of the stability conditions during excavation of the underground openings and the design of the support system to lead the ground stable. This paper has presented a quantitative analysis for determining the final convergence and evaluating the stability of tunnel. The solution has been derived based on the assumptions that the ground is modelled with the Mohr-Coulomb material and the circular plastic area is developed around the circular cavity. Comparison of the results obtained with a numerical tool will predict a more reliable solution for the stability of cavity. The results by the analytical solution and the numerical simulation have demonstrated the displacement confinement by the supporting elements.

(1) A method was proposed that focuses on the change in the cross-section area as a quantity to represent displacement/deformation near the face. It can be concluded that this method provides an approximation of the theoretical result if only the displacements of three points are given.

(2) A comparison of the result of rock bolt element only with those in which the result of shotcrete and steel supports installed both shows that the latter is more effective in reducing wall displacement. This reduction of displacement inward when three supports installed simultaneously was smaller than the sum of the effects of each.

Needless to mention, the ground is not homogeneous, and the stress state is not isotropic. In reality, to estimate/evaluate the effect of lining and the state of stress in the ground, displacement measurement will be performed. Further work is required how to determine the values of the outputs related to the radial pressures and to incorporate displacement measurement on site.

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